MODEL MATHEMATICS CLASS: XII

Time: 3Hours Max.Marks:100

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B, C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of 4 marks each and section C comprises of o7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION A

1. Let E={1,2,3,4} and F={1,2}. Then find the number of onto functions from E to F.
2. Find the numerical value of $\tan[2\tan^{-1}(1/5)-\pi/4]$
3. For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse?
4. If A=$\begin{bmatrix} \cos\propto & -\sin\propto \\ \sin\propto & \sin\propto \end{bmatrix}$ then $A+A^T=I$, Find the value of $\propto$.
5. Give an example of two nonzero 2x2 matrices A,B such that AB=O
6. Evaluate: $\int \frac{1+\cot x}{x+\log\sin x}\,dx$
7. Evaluate: $\int_{-1}^{1} f(x)dx$ where f(x) = x-[x]; [x] is the integral part of x.

8. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of magnitudes 3,4,5 respectively . Let $\vec{a}, be\ perpendicular\ to\ \vec{b}+\vec{c},\ \vec{b}\ to\ \vec{c}+\vec{a}, \vec{c}\ to \vec{a}+\vec{b}.$ Then, find $|\vec{a}+\vec{b}+\vec{c}|$
9. The points with position vectors $60\hat{\imath}+3\hat{\jmath}$,$40\hat{\imath}-8\hat{\jmath}, a\hat{\imath}-52\hat{\jmath}$ are collinear. Find the value of a
10. Find the equation of the line parallel to x axis and passing through the origin

SECTION B

11. Let a relation R on the Set N of natural numbers be defined as (x,y)εR if and only if $x^2-4xy+3y^2=0$ for all x,yεN. Verify that R is reflexive but not symmetric and transitive.

12. Find the value of: $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\frac{1}{8}$

OR

Find x if $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$

13. Show that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

14. Find the values of a and b so that the function

$$f(x)=\begin{cases} x+a\sqrt{2}, & 0\le x\le \frac{\pi}{4} \\ 2xcotx+b, & \frac{\pi}{4}<x\le\frac{\pi}{4} \\ acos2x-bsinx, & \frac{\pi}{2}<x\le\pi \end{cases}$$ is continuous for 0≤x≤π

15. If cos y =x cos(a+y), with cos a ≠±1, prove that $\frac{dy}{dx}=\frac{cos^2(a+y)}{sina}$

OR

If x=a(t+sint) and y=a(1+cost) . Find $\frac{d^2y}{dx^2}\ at\ t=\frac{\pi}{2}$

16. Find the intervals in which $xe^{x(1-x)}$ is strictly increasing or decreasing .

17. Evaluate $\int \frac{x+1}{x(1+xe^{x})^{2}}\,dx$

OR

Evaluate $\int \sqrt{1+\sin\left(\frac{x}{2}\right)}\,dx$

18. Form the differential equation of the function $(a+bx)e^{y/x} = x$

OR

Form the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes.

19. Solve the differential equation $[x \sin^2(y/x)-y]\,dx+xdy = 0$

20. For any two vectors $\vec{a}$ and $\vec{b}$

$$prove\ that\ (1+|\vec{a}|^2)\left(1+\left|\vec{b}\right|^2\right) = \left|1-\vec{a}.\vec{b}\right|^2 + \left|\vec{a}+\vec{b}+(\vec{a}\times\vec{b})\right|^2$$

21. State whether the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect or not. If intersecting find the point of intersection.

22. There are 5 cards numbered 1 to 5. One number on one card. Two cards are drawn at random without replacement. Find the probability distribution of the sum of the numbers on the two cards.

SECTION C

23. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find A^{-1} and hence solve the system of linear equations

X+2y+z=4 ,-x+y+z=0, x-3y+z=2

OR

Using elementary transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

24. A window of perimeter(including the base of the arc) is in the form of a rectangle surrounded by a semi circle. The semi-circular portion is fitted

with coloured glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much lighrt per square metre as the coloured glass does. Show that the ratio of the length and breadth of the rectangle is 6:6+π, sothat the window transmits maximum light.

25. Sketch the region bounded by the curves y=$\sqrt{5-x^2}$ and y=$|x-1|$ and find its area

26. Evaluate $\int_0^{\pi} \frac{x\sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx$

OR

Evaluate $\int_{-1}^{3/2} |x\sin\pi x| dx$

27. Find the equation of the plane passing through the point(1,1,1) and containing the line
$\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k})\lambda(3\hat{\imath} - \hat{\jmath} + 5\hat{k})$. Also, show that the plane contains the line
$\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$

28. Every gram of wheat provides 0.1 gm of proteins and 0.25gm of carbohydrates. The corresponding values for rice are 0.05gm and 0.5gm respectively. Wheat costs Rs.4 per kg and rice Rs.6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50gms and 200gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an LPP and solve it graphically.

29. Bag A contains 3 red and 4 black balls and bag B contains 4 red and 5 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.