CBSE Math Sample Paper Class 12 2012

M.M : 100 **TIME: 3hr**

GENERAL INSTRUCTION:

- ***(a) All questions are compulsory.***
- ***(b) This question paper consists of 29 questions divided into three section A, B, and C. Section A comprises of 10 question of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.***
- ***(c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.***
- ***(d) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.***
- ***(e) Use of calculators is not permitted. You may ask for logarithmic tables, if required.***

SECTION-'A'

Question number 1 to 10 carry 1 mark each.

Q1) Prove that: $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$.

Q2) If $\sin\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{x}\right) = 1$, then find the value of x.

Q3) Find the matrix X, such that: $2X - \begin{bmatrix} 4 & -3 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 0 & 6 \end{bmatrix}$.

Q4) For any two non zero vectors $\vec{a}\ and\ \vec{b}$, show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$.

Q5) If $\vec{a}$ and $\vec{b}$ are represented along the two diagonals of a parallelogram, then write the area of a parallelogram in the terms of $\vec{a}\ and\ \vec{b}$.

Q6) If $\vec{a}$ is a unit vector and $(\vec{x} + \vec{a}).(\vec{x} - \vec{a}), find\ |\vec{x}|$

Q7) The total revenue received from the sale of shirts, advertising 'Keep Your Environment Clean' is given by the function , $R(x) = x^2 + 4x + 11$.If the marginal revenue is defined as the rate of change of

R(x) with respect to the number of shirts sold at an instant, find the marginal revenue when 5 shirts are sold.

Q8) Write the order of the differential equation: $y - px = \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$.

Q9) Write the value of x – 2y + 3z, if$\begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Q10) Given a square matrix A of order 3, such that $|A'| = 15, find\ |A.adjA|$

SECTION-'B'

Question number 11 to 22 carry 4 marks each.

*Q11) Let A represents a set of all students, who would like to be a part of 'CLEANLINESS DRIVE'. If R be the relation in set A, given by x R y implies student x and student y are associated with the 'CLEANLINESS DRIVE', show that the relation R is an equivalence relation. Do you want to be a part of the drive? Why?

Q12) Prove that: $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$.

OR

If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$,then prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$.

*Q13)Two schools A and B decided to award prizes to their students for three values honesty(x),punctuality(y) and empathy(z).School A decided to award a total of Rs.22,000 for the three values to 10,8,and 6 students respectively, while school B decided to award Rs. 21,400 for three values to 8,6,10 students respectively. If all the three prizes amount to Rs.5, 400 then,

(i) Represents the above situation by a matrix equation and form linear equations using matrix multiplication.
(ii) Is it possible to solve the system of equations so obtained using matrices?
(iii) Which value do you prefer to be rewarded most and why?

Q14) Express the equation $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ in terms of an equation involving inverse trigonometrical functions and hence prove that,$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Q15) Iflog$(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right), then\ prove\ that\ \frac{dy}{dx} = \frac{x+y}{x-y}$.

Q16) Is the function f defined as $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x=0?

Q17) Evaluate: $\int (6x-5)\sqrt{x^2+x+1}\,dx$.

OR

Evaluate: $\int \frac{1}{\cos(x+\alpha)\sin(x+\beta)}dx$

Q18) Evaluate: $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$.

Q19) Evaluate: $\int e^x \left(\frac{x^2+1}{(x+1)^2}\right) dx$.

Q20) Find λ, so that the four points with position vectors $-6\hat{\imath}+3\hat{\jmath}+2\hat{k}, 3\hat{\imath}+\lambda\hat{\jmath}+4\hat{k}, 5\hat{\imath}+7\hat{\jmath}+3\hat{k}$ *and* $-13\hat{\imath}+17\hat{\jmath}-\hat{k}$ are coplanar.

Q21) Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k})=1$ *and* $\vec{r}.(2\hat{\imath}+3\hat{\jmath}-\hat{k})+4=0$ and parallel to x- axis.

OR

Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of $3\sqrt{2}$ units from the point (1, 2, 3)

*Q22) Three persons A,B and C throw a die in succession till one of them gets a 'six' and wins the ticket to be a part of the group 'TOWARDS A JUST SOCIETY'. Find their respective probabilities of winning, if A starts followed by B and C. If C selected for the group, then C should play at which turn? Write two lines about 'TOWARDS A JUST SOCIETY".

SECTION-'C'

Question numbers 23 to 29 carry 6 marks each.

Q23) If A $=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ *and* $B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, find AB and hence solve the equations,

2x-y+z=-1,-x+2y-z=4, x-y+2z=-3.

OR

Using properties of determinants prove that:

$$\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}.$$

*Q24) In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probably that he copies answer is $\frac{1}{6}$.The probability that his answer is correct given that he copied it is $\frac{1}{8}$.Find the probability that he knew the answer to the question ,given that he correctly answered it.

A student does not know the answer to one of the questions in a test .The evaluation process has a negative marking.

Which value would a student violate if he uses unfair means? How would an act like the above hamper his character development in the coming years?

*Q25) If a young man rides his motorcycle at 25km/h, he has to spend Rs.2 per km on petrol. If he rides at a faster speed of 40km/h, the petrol cost increase at Rs.5 per km.He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel in one hour .Express this as an L.P.P and solve it graphically. These days we talk about saving our Natural Resources .Why do we need to save petrol? Answer in two line.

*Q26) Find the image of the point (1, 2, 3) in the plane x+2y+4z=38.Write the two life skills, which one must acquire, along the image.

Q27) Using integration, find the area of the region:$\{(x, y): |x + 2| \leq y \leq \sqrt{20 - x^2}\}$.

Q28) Solve the differential equation:$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.

Q29) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle$\alpha\ is\ \frac{4}{27}\pi h^3 \tan^2 \alpha$.

OR

Find the interval for which the function $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is increasing or decreasing in the interval $\left(0, \frac{\pi}{2}\right)$