Steady State Error

Steady state error is the difference between desired output and actual output. It is denoted by e_{ss} and its Laplace transformation is denoted by E(s). When the reference input is applied to the given system then the information given about the level of desired output is observed. The actual output is feed back to the input side and it is compared with the input signal. Thus steady state error can also be defined as the difference between the reference input and the feedback signal. This post includes Control system notes on Steady State Error.

Derivation of Steady state error:

Consider a simple closed loop system as shown in figure below:

Different notation used

R(s)= Laplace transformation of input, r(t)

B(s)= Laplace transformation of feedback signal, b(t)

E(s)= Laplace transformation of error signal, e(t)

H(s)= Laplace transformation of feedback element,

C(s)= Laplace transformation of output signal, c(t)

NOTE: Here negative feedback is used.

E(s) = R(s) - B(s)

But, B(s) = C(s). H(s), on putting the value in E(s) we get,

Therefore, E(S) = R(s) – C(s). H(s)

Also, C(s) = G(S).E(s)

E(s) = G(s) – G(s) E(s) H(s)

Therefore, E(s) + G(s) E(s) H(s) = R(s)

Thus we get, E(s)[1 + G(s) H(s)] = R(s)

Therefore, E(s) = R(s) / 1 + G(s) H(s)

If positive feedback is used the expression is

Combining E(s) = E(s)/ 1 – G(s) H(s)

On combining we get

E(s) = E(s)/ 1 ± G(s) H(s)

For a unity feedback system H(s) = 1 then

E(s) = R(s)/ 1 + G(s) or E(s) = R(s)/ 1 - G(s)

Here E(s) is the error signal in Laplace domain. In the time domain error signal is denoted by e(t) and it is obtained by taking Laplace inverse of E(s). That means,

$e(t) = L^{-1} E(s)$

Now the steady state error is denoted by e_{ss} and it is given by,

Steady State Error= $\lim_{t\to\infty} e(t)$

We will make use of final value theorem in Laplace transform. If Laplace transform of time domain signal is f(t) then according to final value theorem,

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Applying this theorem to the equation of steady state error we get,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

On putting the value of E(s) in above equation

$$e_{ss} = \lim_{s\to o} \left(\frac{sR(s)}{1+G(S)H(s)}\right)$$

Effect of Input on Steady state error:

The of Input- R(s)

Type of system- G(s) H(s)

The Steady state error is calculated for three types of input : step, ramp, parabolic. The equation obtained of e_{ss} is valid for any input R(s), hence it will be used for these inputs.

Static error coefficient:

The response that remain after the transient response has died out is called steady state response. The steady sate response is important to find the accuracy of the output. The difference between steady state response and desired response gives the steady state error.

The control system has following steady state errors for change in positions, velocity and acceleration.

K_p = Positional error constant

K_v = Velocity error constant

K_a = Acceleration error constant

These constants are called static error coefficient. They have the ability to minimize the steady error.

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